ОРМО
Открытая региональная межвузовская олимпиада

Общий балл	Дата	Ф.И.О. Жюри	Подпись
69б		Червоненкая АС	

Шифр 041873

1) Дано:

S

А) $\frac{1}{3}S^{(1)}: a = const\ (a_A > 0)$

$\frac{2}{3}S^{(2)}: v = const$

$\frac{1}{3}S^{(3)}: a = const\ (a_A < 0)$

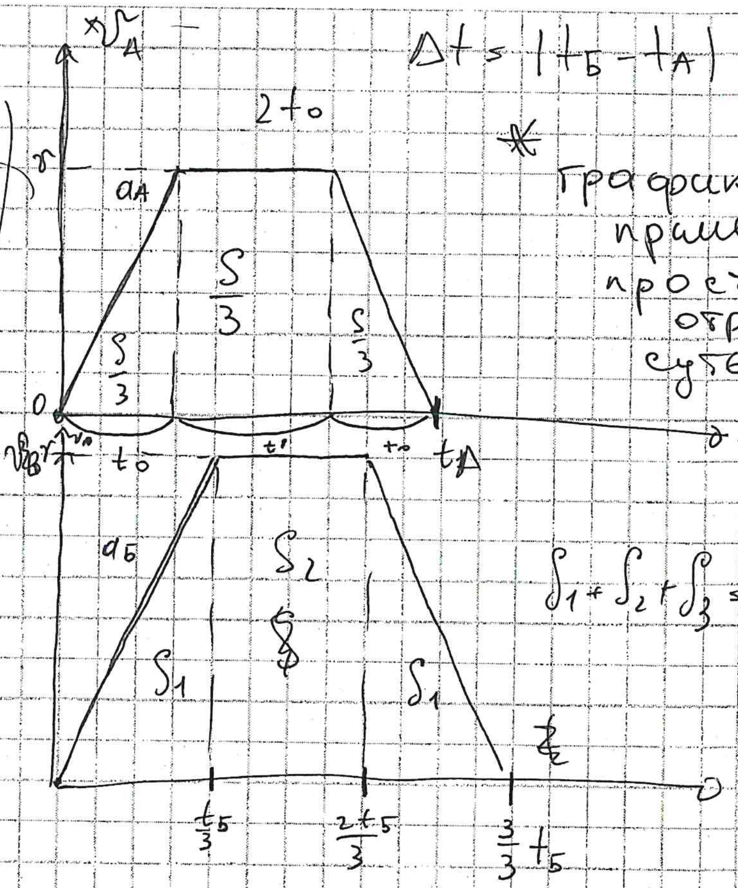
Б) $\frac{1}{3}t: a = const\ (a_Б > 0)$

$\frac{1}{3}t: v = const$

$\frac{1}{3}t: a = const\ (a_Б < 0)$

$\Delta t - ?$

$\Delta t = |t_Б - t_A|$



* графики примерные, просто отражают суть процесса

$S_1 + S_2 + S_3 = S$

$\frac{S}{3} = \frac{a_A t_0^2}{2}, \quad \frac{S}{3} = v t', \quad \frac{S}{3} = \frac{a_A t_0^2}{2}$

$x = v_0 t + \frac{at^2}{2}$

$x = \frac{v_k^2 - v_0^2}{2a}$

$\frac{S}{3} = \frac{v^2}{2a_A} \Rightarrow a_A = \frac{3v^2}{2S}$

$\frac{S}{3} = \frac{t_0^2}{2} \cdot \frac{3v^2}{2S} \Rightarrow \frac{4S^2}{9} = v^2 t_0^2$

$\frac{4S^2}{9v^2} = t_0^2 \Rightarrow t_0 = \frac{2S}{3v}$

$\frac{S}{3} = vt' \Rightarrow t' = \frac{S}{v \cdot 3}$

$t_A = 2t_0 + t' = 2 \cdot \frac{2S}{3v} + \frac{S}{3v} = \frac{4S}{3v} + \frac{S}{3v} = \frac{5S}{3v} \Rightarrow t_A = \frac{5S}{3v}$

$t_A < t_Б$, значит Антон придет быстрее

$\Delta t = \frac{6S}{3v} - \frac{5S}{3v} = \frac{S}{3v}$

Ответ: $\Delta t = \frac{S}{3v}$

Пусть $\frac{t_Б}{3} = t_2$

$S_2 = v\frac{t_Б}{3}$

$\frac{6S}{3v} = t_Б$

$S_1 = \frac{v^2}{2a_Б}; \quad S_1 = \frac{a_Б t_2^2}{2} \Rightarrow$

$\frac{v^2}{2a_Б} = \frac{a_Б t_2^2}{2}$

$\frac{v^2}{t_2^2} = a_Б^2 \Rightarrow \frac{v}{t_2} = a_Б$

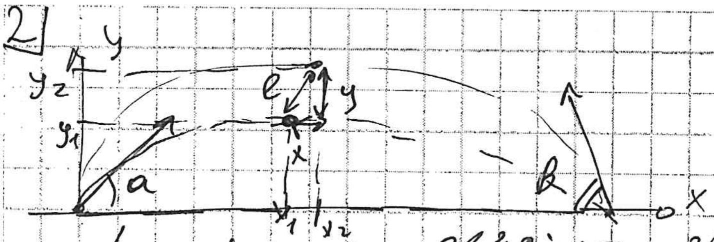
$S_1 = \frac{v^2}{2v}t_2 = \frac{v}{2}t_2 = \frac{vt_Б}{6}$

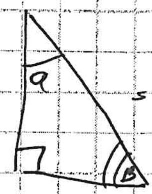
$S = \frac{vt_Б}{2}$

12б

$S = 2S_1 + S_2 = \frac{vt_Б}{6} + \frac{2vt_Б}{2 \cdot 3} = \frac{3vt_Б}{6} = \frac{vt_Б}{2}$

2)





$\Rightarrow \sin a = \cos\beta$

$\cos a = \sin\beta$

$l_1 = l_2 = l = \frac{v_0^2 \sin 2a}{g} = \frac{v_0^2 \sin 2\beta}{g} \Rightarrow \sin 2a = \sin 2\beta \Rightarrow$

$\Rightarrow 2\beta = 180 - 2a \Rightarrow$

$\Rightarrow \beta = 90 - a$

$x_1 = v_0 \cos a t$

$x_2 = S - v_0 \cos\beta t = S - v_0 t \sin a$

$y_1 = v_0 \sin a t - \frac{gt^2}{2}$

$y_2 = v_0 \cos a t - \frac{gt^2}{2}$

$x = x_2 - x_1 = S - v_0 t \sin a - v_0 t \cos a =$

$= S - v_0 t(\cos a + \sin a)$

! $\sin^2 a + \cos^2 a = 1$

$y = y_2 - y_1 = v_0 t(\cos a - \sin a)$

$x^2 = S^2 - 2v_0 t S(\cos a + \sin a) + v_0^2 t^2 (\cos a + \sin a)^2 =$

$= S^2 - 2v_0 t S\cos a - 2S v_0 t \sin a + \underline{v_0^2 t^2 \cos^2 a} + \underline{v_0^2 t^2 \sin^2 a} + 2v_0^2 t^2 \sin a \cos a =$

$= S^2 + v_0^2 t^2 - 2v_0 t S(\cos a + \sin a) + 2v_0^2 t^2 \sin a \cos a$

$y^2 = v_0^2 t^2 (\cos^2 a - 2\cos a \sin a + \sin^2 a) = v_0^2 t^2 (1 - 2\cos a \sin a)$

$x^2 + y^2 = S^2 + v_0^2 t^2 - 2v_0 t(\cos a + \sin a)S + \cancel{2v_0^2 t^2 \sin a \cos a} + v_0^2 t^2 - \cancel{2v_0^2 t^2 \cos a \sin a}$

$x^2 + y^2 = S^2 + 2v_0^2 t^2 - 2v_0 t(\cos a + \sin a)S$

$l = \sqrt{x^2 + y^2}$, пусть $x^2 + y^2 = \varphi$

l_{min} когда $(x^2+y^2)_{min} \Rightarrow$ когда $\varphi_{min} \Rightarrow \frac{d\varphi}{dt} = 0$

$\frac{d\varphi}{dt} = \cancel{2}v_0^2 2t - \cancel{2}v_0 S(\cos a + \sin a) = 0$

$2v_0^{\cancel{2}} t = \cancel{v_0} S(\cos a + \sin a)$

$2v_0 t = S(\cos a + \sin a)$

$t_0 = \frac{S}{2v_0}(\cos a + \sin a)$ — время когда l_{min}

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$$x = S - \cancel{v_0}(\cos a + \sin a) \cdot \frac{S}{2\cancel{v_0}}(\cos a + \sin a) =$$

$$= S - \frac{S}{2}(\cos a + \sin a)^2 = \frac{S}{1}\left(1 - \frac{1}{2}(\cos^2 a + \sin^2 a + 2\sin a\cos a)\right) =$$

$$= S\left(1 - \frac{1}{2}(1 + 2\sin a\cos a)\right) =$$

$$= \frac{S}{2}(2 - 1 - 2\sin a\cos a) = \frac{S}{2}(1 - 2\sin a\cos a)$$

$$y = \cancel{v_0}\frac{(\cos a - \sin a)S}{2\cancel{v_0}}(\cos a + \sin a) = \frac{S}{2}(\cos^2 a - \sin^2 a)$$

$$x^2 = \frac{S^2}{4}(1 - 4\sin a\cos a + 4\sin^2 a\cos^2 a)$$

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$$y^2 = \frac{S^2}{4}(\cos^4 a - 2\cos^2 a\sin^2 a + \sin^4 a)$$

$$l^2 = x^2 + y^2 = \frac{S^2}{4}(1 - 4\sin a\cos a + 4\sin^2 a\cos^2 a + \cos^4 a - 2\cos^2 a\sin^2 a + \sin^4 a).$$

$$= \frac{S^2}{4}(1 - 4\sin a\cos a + \cos^4 a + 4\sin^2 a\cos^2 a + \sin^4 a - 2\cos^2 a\sin^2 a) =$$

$$= \frac{S^2}{4}(1 - 4\sin a\cos a - 2\cos^2 a\sin^2 a + \cos^2 a + \sin^2 a)$$

$$= \frac{S^2}{4}(1 - 4\sin a\cos a + \cos^4 a + \sin^4 a + 2\sin^2 a\cos^2 a + \cancel{2\sin^2 a\cos^2 a} -$$

$$- \cancel{2\cos^2 a\sin^2 a}) = \frac{S^2}{4}(1 - 4\sin a\cos a + \underbrace{(\cos^2 a + \sin^2 a)^2}_{1}) =$$

$$= \frac{S^2}{4}(2 - 4\sin a\cos a) = \frac{S^2}{2}(1 - 2\sin a\cos a)$$

Из баллистики: $L = \frac{v_0^2\sin 2a}{g} = \frac{2v_0^2\sin a\cos a}{g} \Rightarrow S = \frac{v_0^2}{g} \cdot 2\sin a\cos a \Rightarrow$

$$\Rightarrow \boxed{\frac{Sg}{v_0^2} = 2\sin a\cos a} \quad l^2 = \frac{S^2}{2}\left(1 - \frac{Sg}{v_0^2}\right)$$

Ответ

$$l^2 = \frac{S^2}{2}\left(\frac{v_0^2 - Sg}{v_0^2}\right) \Rightarrow \boxed{l = \frac{S}{v_0}\sqrt{\frac{v_0^2 - Sg}{2}}}$$

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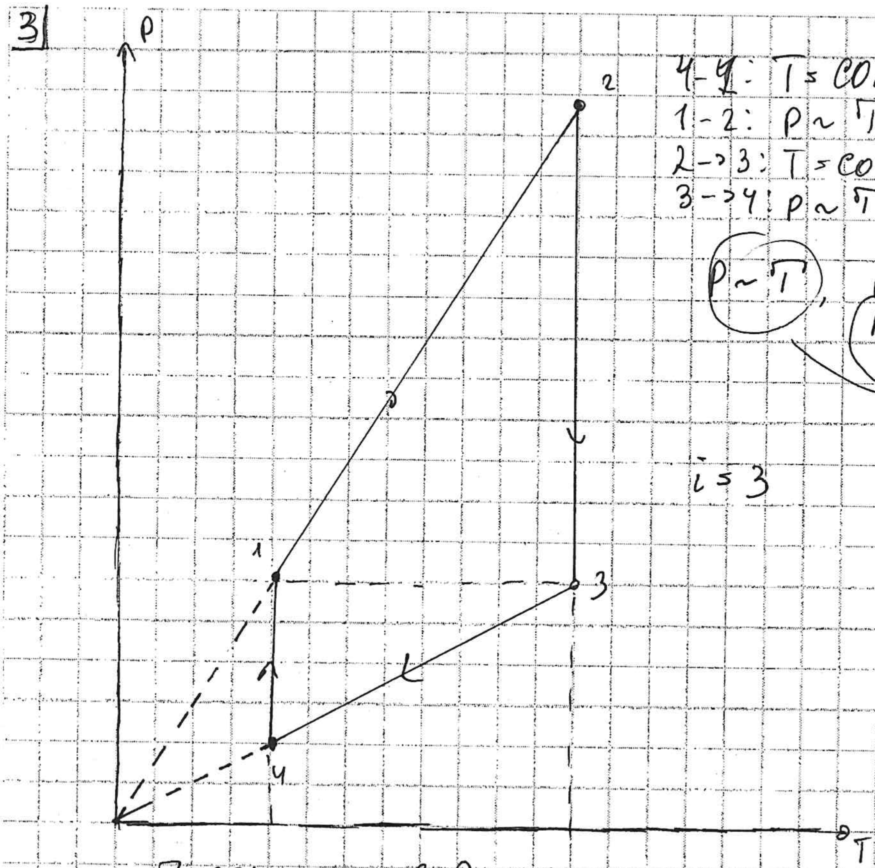


4-1: $T = const$

1-2: $P \sim T \Rightarrow V = const$

2->3: $T = const$

3->4: $P \sim T \Rightarrow V = const$

$PV \sim T$

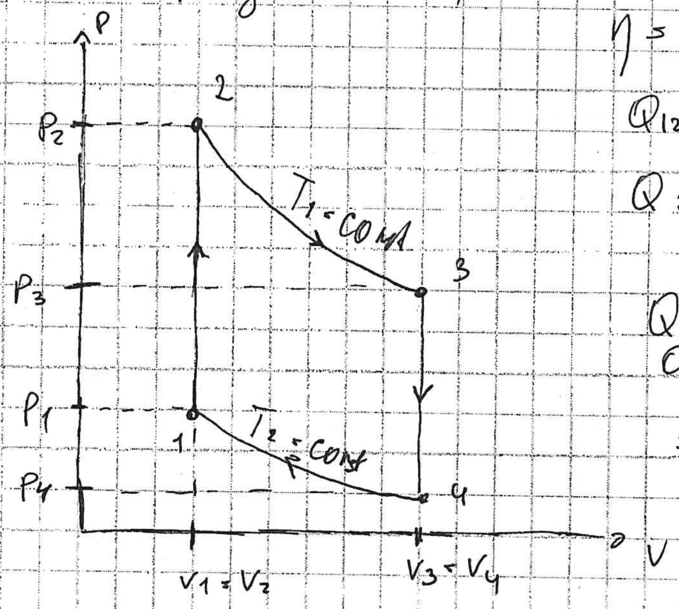
$P \sim T$, $PV = \nu R T$

$P = \frac{\nu R}{V} \cdot T \Rightarrow \frac{\nu R}{V} = const \Rightarrow$

$\Rightarrow V = const$

$i = 3$

Перерисуем в P(V):



$\eta = \frac{A}{Q_н} = 1 - \frac{Q_х}{Q_н}$

Q_{12}, Q_{41} – нагр

Q_{23}, Q_{34} – холод

$Q_{12} = \Delta U_{12} + A_{12}$, $A_{12} = 0$, т.к. $V = const$

$Q_{12} = \Delta U_{12} = \frac{3}{2}\nu R(T_1 - T_2) = \frac{3}{2}\nu R T_1 - \frac{3}{2}\nu R T_2 =$

$= \frac{3}{2}(P_2V_1 - P_1V_1) = \frac{3}{2}V_1(P_2 - P_1)$

$Q_{34} = \frac{3}{2}V_3(P_4 - P_3)$

$Q_{23} = \Delta U_{23} + A_{23}$, $\Delta U_{23} = 0$, т.к. $T = const$

$Q_{23} = A_{23} = \int_{V_2}^{V_3} P\,dV = \int_{V_2}^{V_3} \frac{\nu R T_1}{V}\,dV = \nu R T_1 \int_{V_2}^{V_3} \frac{dV}{V} = \nu R T_1 \ln\frac{V_3}{V_2}$

$Q_{41} = \Delta U_{41} + A_{41}$, $\Delta U_{41} = 0$, т.к. $T = const \Rightarrow$

$\Rightarrow Q_{41} = A_{41} = \int_{V_4}^{V_1} \nu R T_2 \cdot \frac{dV}{V} = \nu R T_2 \ln\frac{V_1}{V_4}$

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$Q_{12} = \frac{3}{2}V_1(P_2-P_1) = \frac{3}{2}(\nu RT_2 - \nu RT_1) = \frac{3}{2}\nu R(T_1 - T_2)$

$Q_{23} = \nu RT_1 \ln\frac{V_3}{V_2}$

$Q_{34} = \frac{3}{2}V_3(P_4-P_3) = \frac{3}{2}(\nu RT_2 - \nu RT_1) = \frac{3}{2}\nu R(T_2 - T_1)$

$Q_{41} = \nu RT_2 \ln\frac{V_1}{V_4}$

$P_2 = 18P_0 \quad P_3 = 6P_0$

$P_1 = 6P_0 \quad P_4 = 2P_0$

$T_1 = 4T_0$

$T_2 = 12T_0$

$PV = \nu RT$. при $T=const$:

$\cancel{P_1V_1 = P_2V_2}$ $P_2V_2 = P_3V_3$ $\quad \cancel{P_3V_3}$ $P_4V_4 = P_1V_1$

$\frac{P_2}{P_3} = \frac{V_3}{V_2} \qquad \frac{P_4}{P_1} = \frac{V_1}{V_4}$

$Q_{23} = \nu RT_1 \ln\frac{P_2}{P_3} = A_{23}$

$Q_{41} = \nu RT_2 \ln\frac{P_4}{P_1} = A_{41}$

$A_\Sigma = A_{23} + A_{41} = \nu R\left(T_1\ln\frac{P_2}{P_3} + T_2\ln\frac{P_4}{P_1}\right)$

$Q_{н\Sigma} = Q_{12} + Q_{41} = \frac{3}{2}\nu R(T_1 - T_2) + \nu RT_2\ln\frac{P_4}{P_1} =$

$= \nu R\left(\frac{3}{2}(T_1 - T_2) + T_2\ln\frac{P_4}{P_1}\right)$

$\eta = \frac{\nu R\left(T_1\ln\frac{P_2}{P_3} + T_2\ln\frac{P_4}{P_1}\right)}{\nu R\left(\frac{3}{2}(T_1-T_2) + T_2\ln\frac{P_4}{P_1}\right)} = \frac{4T_0\ln 3 - 12T_0\ln 3}{\frac{3}{2}T_1 - \frac{3}{2}T_2 + T_2\ln 3} =$

$\ln\frac{P_2}{P_3} = \ln\frac{18P_0}{6P_0} = \ln\frac{3P_0}{P_0} = \ln 3$

$= \frac{4\ln 3(T_0 - 3T_0)}{6T_0 - 18T_0 - 12T_0\ln 3} =$

$\ln\frac{P_4}{P_1} = \ln\frac{P_4}{P_1} = \ln\frac{2P_0}{6P_0} = \ln\frac{1}{3} = \ln 3^{-1} = -\ln 3$

$= \frac{(4\ln 3)\cdot(-2T_0)}{-12T_0 - 12T_0\ln 3} =$

$\frac{3}{2}T_1 = \frac{3}{2}\cdot 4T_0 = 6T_0$

$\frac{3}{2}T_2 = \frac{3}{2}\cdot 12T_0 = 18T_0$

$= \frac{-8T_0\ln 3}{-12T_0(1+\ln 3)} = \frac{2\ln 3}{3(1+\ln 3)}$ ответ

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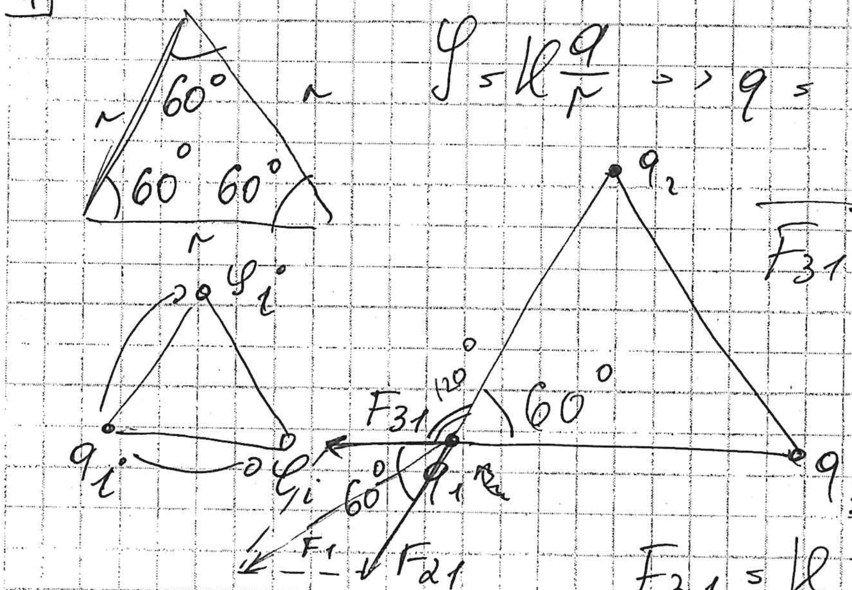


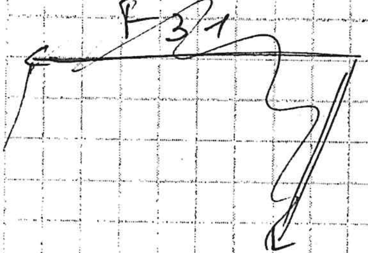
$\varphi = k\frac{q}{r} \Rightarrow q = \frac{\varphi r}{k} \Rightarrow q_i = \frac{\varphi_i r}{k}$

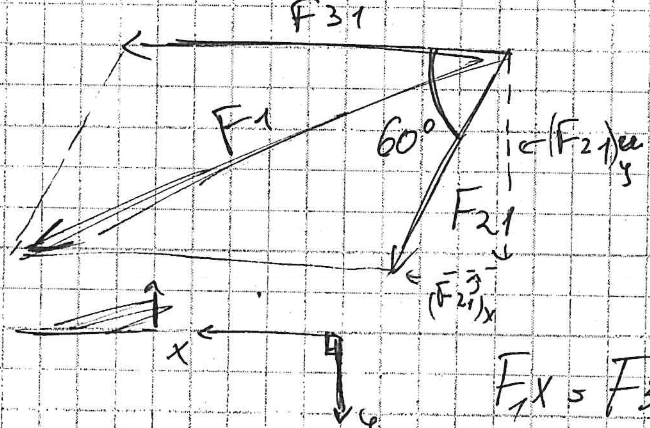
$\vec{F}_{31} + \vec{F}_{21} = \vec{F}_1$

$F_{кл} = k\frac{q_a q_b}{r^2}$

$F_{31} = k\frac{q_3 q_1}{r^2} = \frac{k}{r^2}\cdot\frac{\varphi_3 r}{k}\cdot\frac{\varphi_1 r}{k} = \frac{\varphi_3\varphi_1}{k}$

по аналогии: $F_{21} = \frac{\varphi_2\varphi_1}{k}$





$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2}$

$F_{1x} = F_{31} + F_{21}\cos 60°$

$F_{1y} = F_{21}\sin 60°$

$F_{1x} = \frac{\varphi_3\varphi_1}{k} + \frac{\varphi_2\varphi_1}{k}\cdot\frac{1}{2} = \frac{\varphi_1}{k}\left(\varphi_3 + \frac{\varphi_2}{2}\right)$

$F_{1y} = \frac{\varphi_2\varphi_1}{k}\cdot\frac{\sqrt{3}}{2} = \frac{\varphi_2\varphi_1\sqrt{3}}{2k}$

$F_1 = \sqrt{\frac{\varphi_1^2}{k^2}\left(\varphi_3 + \frac{\varphi_2}{2}\right)^2 + \frac{\varphi_2^2\cdot 3}{4}\cdot\frac{\varphi_1^2}{k^2}}$

$\cancel{F_1 = \frac{\varphi_1^2}{k^2}\varphi_3^2 + \varphi_1}$

$F_1 = \frac{\varphi_1}{k}\sqrt{\varphi_3^2 + 2\varphi_3\frac{\varphi_2}{2} + \frac{\varphi_2^2}{4} + \frac{\varphi_2^2 3}{4}}$

$F_1 = \frac{\varphi_1}{k}\sqrt{\varphi_3^2 + \varphi_3\varphi_2 + \varphi_2^2} = ?$

ответ

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