

Общий балл	Дата	Ф.И.О. Жюри	Подпись
63			<i>[Signature]</i>
Шифр			041918

1. ^① Determine who finishes first.

We have the total time for Anton to complete the race $T_A = \frac{5s}{3v}$, and for Boris

$$T_B = \frac{3s}{2v} \quad \text{and}$$

To compare T_A and T_B , we find a common - ~~denomin~~ denominator.

$$\text{Rewrite } T_A = \frac{5s}{3v} = \frac{10s}{6v} \quad \text{and} \quad T_B = \frac{3s}{2v} = \frac{9s}{6v}$$

Since $s > 0$ and $v > 0$ and $\frac{10s}{6v} > \frac{9s}{6v}$, which means $T_A > T_B$. So Boris will finish first.

$$\text{Anton: using } v^2 = 2a \cdot \frac{s}{3}, \text{ we find } a = \frac{3v^2}{2s}$$

$$\text{Time taken: } t_{A1} = \frac{v}{a} = \frac{2s}{3v}$$

$$\text{Middle } \frac{s}{3}: \text{ Time taken } t_{A2} = \frac{s}{3v}$$

$$\text{Final } \frac{s}{3}: \text{ Time taken } t_{A3} = \frac{2s}{3v}$$

$$\text{Total time: } T_A = t_{A1} + t_{A2} + t_{A3} = \frac{2s}{3v} + \frac{s}{3v} + \frac{2s}{3v} = \frac{5s}{3v}$$

$$\text{Boris: First } \frac{T_B}{3} = v = a \cdot \frac{T_B}{3} \quad \text{get } a = \frac{3v}{T_B}$$

$$S_{B1} = \frac{1}{2} a \left(\frac{T_B}{3} \right)^2 = \frac{v T_B}{6}$$

$$\text{Middle } \frac{T_B}{3} = S_{B2} = \frac{v T_B}{3}$$

$$\text{Final } \frac{T_B}{3} = S_{B3} = \frac{v T_B}{6}$$

$$\text{Total Distance} = S = S_{B1} + S_{B2} + S_{B3}$$

$$= \frac{v T_B}{6} + \frac{v T_B}{3} + \frac{v T_B}{6}$$

$$= \frac{2v T_B}{3}$$

$$\text{Solving for } T_B = T_B = \frac{3s}{2v}$$

Общий балл	Дата	Ф.И.О. Жюри	Подпись
Шифр			

2. Establish the equations of motion Left Ball (θ_1) $\Downarrow$

① In the horizontal direction, the velocity component is $v_0 \cos \theta_1$. According to the formula for uniform linear motion $x = vt$ the horizontal position is $x_1(t) = v_0 \cos \theta_1 \cdot t$. In the vertical direction, it is a ~~vert~~ vertical - upward - throwing motion. The initial velocity component is $v_0 \sin \theta_1$. According to the formula for uniformly variable linear motion $x = vt + \frac{1}{2}at^2$ (where $a = -g$), the vertical position is $y_1(t) = v_0 \sin \theta_1 \cdot t - \frac{1}{2}gt^2$.

Right Ball (θ_2)

② Position as a function of time t (thrown to the left)

$$\begin{cases} x_2(t) = S - v_0 \cos \theta_2 \cdot t \\ y_2(t) = v_0 \sin \theta_2 \cdot t - \frac{1}{2}gt^2 \end{cases}$$

③ Derive the distance formula between the two balls. According to the distance formula for two points in space, the square of the distance between the two balls.

$$D^2(t) \text{ is } D^2(t) = [S - v_0 t (\cos \theta_1 + \cos \theta_2)]^2 + [v_0 t (\sin \theta_2 - \sin \theta_1)]^2$$

Find the time t corresponding to the minimum distance.

After differentiation and calculation, we get $t = \frac{S(\cos \theta_1 + \cos \theta_2)}{v_0[(\cos \theta_1 + \cos \theta_2)^2 + (\sin \theta_2 - \sin \theta_1)^2]}$

④ Calculate the square of the minimum distance $D^2_{\min}$

Substitute the above - obtained t into $D^2(t)$ and simplify to get:

$$D^2_{\min} = S^2 \frac{(\sin \theta_2 - \sin \theta_1)^2}{(\cos \theta_1 + \cos \theta_2)^2 + (\sin \theta_2 - \sin \theta_1)^2}$$

⑤ Simplify using trigonometric identities.

We can finally get the minimum distance $D_{\min} = S \cdot \left| \sin \left(\frac{\theta_1 - \theta_2}{2} \right) \right|$

In conclusion, the minimum distance between the two balls during their flight is

$$S \cdot \left| \sin \left(\frac{\theta_1 - \theta_2}{2} \right) \right|$$

100

Общий балл	Дата	Ф.И.О. Жюри	Подпись
Шифр			

3. Analyze the cyclic process

Process 1-2 : From the diagram we know that this is an isochoric process (since no specific coordinate values are given, assume the temperature increases by ΔT_{12} from state 1 to state 2) According to the heat - absorption formula for the isochoric process $Q_{12} = \frac{3}{2} n R \Delta T_{12}$, the gas absorbs heat in this process.

Process 2-3 = Assume that the pressure is constant from state 2 to state 3, which is an isobaric process. Let the temperature change be ΔT_{23} . According to the heat - absorption formula for the isobaric process $Q_{23} = \frac{5}{2} n R \Delta T_{23}$. the gas absorbs ~~heat~~ heat in this process

Process 3-4. Since there are no coordinates in the diagram, assume it is a heat-release process.

process 4-1 also assume it is a heat-release process

isobaric process $Q_{23} = \frac{5}{2} n R \Delta T_{23}$.

count:

$$1-2 : Q_{12} = \frac{3}{2} n R (T_2 - T_1)$$

$$2-3 : Q_{23} = \frac{5}{2} n R (T_3 - T_2)$$

~~the net work of the cyclic process~~ The net work of the cyclic process

$$W = p_2 (V_3 - V_2) = p_2 \left(\frac{n R T_3}{p_2} - \frac{n R T_2}{p_2} \right) = n R (T_3 - T_2)$$

$$\text{The total heat absorbed } Q_{in} = \frac{3}{2} n R (T_2 - T_1) + \frac{5}{2} n R (T_3 - T_2)$$

The efficiency of the heat engine

$$\eta = \frac{W}{Q_{in}} = \frac{n R (T_3 - T_2)}{\frac{3}{2} n R (T_2 - T_1) + \frac{5}{2} n R (T_3 - T_2)}$$

$$Q_{in} = \frac{3}{2} R (2-1) + \frac{5}{2} R (3-2)$$

$$= \frac{3}{2} R + \frac{5}{2} R$$

$$= 4R$$

$$W = R(3-2) = R$$

$$\eta = \frac{R}{4R} = 0.25 = 25\%$$

185

Общий балл	Дата	Ф.И.О. Жюри	Подпись
Шифр			

4. The resulting force $\vec{F}_1$ acting on the first ball due to the second and third balls is derived by considering the expression implies to: $F_1 = \frac{\phi_1}{k} \sqrt{\phi_2^2 + \phi_3^2 + \phi_2 \phi_3}$.

① Express charges in terms of Potentials.

$$\text{charges } q_1 = \frac{\phi_1 d}{k}, q_2 = \frac{\phi_2 d}{k}, q_3 = \frac{\phi_3 d}{k}$$

② Coulomb's law: $F_{12} = k \frac{q_1 q_2}{d^2} = \frac{\phi_1 \phi_2}{k}, F_{13} = \frac{\phi_1 \phi_3}{k}$

③ Angle between forces: 60° .

Resultant force magnitude

$$F_1 = \sqrt{F_{12}^2 + F_{13}^2 + 2F_{12}F_{13}\cos(60^\circ)} = \frac{\phi_1}{k} \sqrt{\phi_2^2 + \phi_3^2 + \phi_2 \phi_3}$$

Final answer

$$F_1 = \frac{\phi_1}{k} \sqrt{\phi_2^2 + \phi_3^2 + \phi_2 \phi_3} \quad 150$$

Общий балл	Дата	Ф.И.О. Жюри	Подпись
Шифр			

5. Initial conditions (10% relative humidity)

First, find the vapor pressure. The relative humidity is 10%, and the saturated vapor pressure at 43°C is good 9000 Pa. So the vapor pressure $P_v = 0.1 \times 9000 \text{ Pa} = 900 \text{ Pa}$

Then, calculate the dry-air pressure. The total atmospheric pressure is 100000 Pa, so the dry-air pressure $P_{da} = 100000 \text{ Pa} - 900 \text{ Pa} = 99100 \text{ Pa}$

Density of dry air $\frac{99100 \times 0.02897}{8.314 \times 316.15} \approx 1.092 \text{ kg/m}^3$

Density of water Vapor: $\frac{900 \times 0.018015}{8.314 \times 316.15} \approx 0.0067 \text{ kg/m}^3$

Total density: $1.092 + 0.0067 \approx 1.098 \text{ kg/m}^3$

Final conditions.

The vapor pressure $P'_v = 0.9 \times 100000 \text{ Pa} - 8100 \text{ Pa} = 91900 \text{ Pa}$

The density of dry air $\frac{P'_{da} M}{RT} = \frac{91900 \times 0.02897}{8.314 \times 316.15} \approx 1.013 \text{ kg/m}^3$

The density of water Vapor $P'_v = \frac{P'_v M_v}{RT} = \frac{8100 \times 0.018015}{8.314 \times 316.15} \approx 0.0555 \text{ kg/m}^3$

The total density of the air $P_2 = 1.013 + 0.0555 \approx 1.0685 \text{ kg/m}^3$

The buoyant force is given by $F_b = \rho V g$

The initial buoyant force $F_{b1} = \rho_1 V g$
 $= 1.098 \text{ kg/m}^3 \times 5000 \text{ m}^3 \times 9.81 \text{ m/s}^2$
 $\approx 53906 \text{ N}$

The Final buoyant force $F_{b2} = \rho_2 V g$
 $= 1.0685 \text{ kg/m}^3 \times 5000 \text{ m}^3 \times 9.81 \text{ m/s}^2$
 $\approx 52459 \text{ N}$

The difference in buoyant force ΔF_b
 $= F_{b1} - F_{b2}$
 $= 53906 \text{ N} - 52459 \text{ N}$
 $\approx 1447 \text{ N}$

Mass to drop: $\frac{1447 \text{ N}}{9.81 \text{ m/s}^2} \approx 147.5 \text{ kg}$

The difference in air density: 0.030 kg/m^3

The mass difference: $0.030 \text{ kg/m}^3 \times 5000 \text{ m}^3 = 150 \text{ kg}$

Thus, the weight of the ballast that must be dropped is 150 kg

Всего ? 105