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1. Two track and field athletes, Anton (A) and Boris (B), compete in running with a distance of S. Each of them followed their own tactics: Anton ran with constant acceleration for the way, ran with constant velocity v for the second third of the way, and slowed down with the same acceleration for the last ~~tio~~ third of the way. Boris ran at constant acceleration for the first third of the time, ran at constant speed v for the second third of the time (the same as Anton's), and slowed down for the last third of the time with the same acceleration as at the ~~first~~ start. Which of the athletes will finish fish? By what time Δt will he overtake his opponent?

Anton's Strategy

First $\frac{S}{3}$: Using $v^2 = 2a\cdot\frac{S}{3}$, we find $a = \frac{3v^2}{2S}$.

Time taken: $t_{A1} = \frac{v}{a} = \frac{2S}{3v}$

Middle $\frac{S}{3}$: Time taken: $t_{A2} = \frac{S}{3v}$

Final $\frac{S}{3}$: Deceleration $-a$ to rest

Time taken: $t_{A3} = \frac{2S}{3v}$

Total Time: $T_A = t_{A1} + t_{A2} + t_{A3} = \frac{2S}{3v} + \frac{S}{3v} + \frac{2S}{3v} = \frac{5S}{3v}$

Boris's Strategy

First $\frac{T_B}{3}$: $v = a\cdot\frac{T_B}{3}$, get $a = \frac{3v}{T_B}$

$S_{B1} = \frac{1}{2}a\left(\frac{T_B}{3}\right)^2 = \frac{vT_B}{6}$

Middle $\frac{T_B}{3}$: $S_{B2} = \frac{vT_B}{3}$

Final $\frac{T_B}{3}$: ~~+~~ $S_{B3} = \frac{1}{2}(-a)\left(\frac{T_B}{3}\right)^2 = \frac{vT_B}{6}$

$S = S_{B1} + S_{B2} + S_{B3} = \frac{2vT_B}{3}$ $\quad T_B = \frac{3S}{2v}$

$\Delta t = T_A - T_B = \frac{5S}{3v} - \frac{3S}{2v} = \frac{S}{6v}$

~~Conclusion~~

18Б.

v = ?

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2. Two jugglers simultaneously throw balls to each other with the same initial velocity v_0, but at different angles to the horizon. Detemine the minimum distance between the balls dusing the flight. The distance between the jugglers is S. Balls are thrown and caught at the same height. Ignore the air ~~restr~~ resistance.

Left Ball (θ_1)

$$\begin{cases} x_1(t) = v_0 \cos\theta_1 \cdot t \\ y_1(t) = v_0 \sin\theta \cdot t - \frac{1}{2} g t^2 \end{cases}$$

Right Ball (θ_2)

$$\begin{cases} x_2(t) = S - v_0 \cos\theta_2 \cdot t \\ y_2(t) = v_0 \sin\theta_2 \cdot t - \frac{1}{2} \end{cases}$$

The squared distance between the two balls is:

$$D^2(t) = [S - v_0 t(\cancel{t_1}\cos\theta_1 + \cos\theta_2)]^2 + [v_0 t(\sin\theta_2 - \sin\theta_1)]^2$$

To find the minimum distance, we differentiate $D^2(t)$ with respect to t and set the derivative to ~~zor~~ zero. Solving for t gives:

$$t = \frac{S(\cos\theta_1 + \cos\theta_2)}{v_0 [(\cos\theta_1 + \cos\theta_2)^2 + (\sin\theta_2 - \sin\theta_1)^2]}$$

Substituting t into $D^2(t)$ and simplifying, we obtain the squared minimum distance: $D^2_{min} = S^2 \cdot \frac{(\sin\theta_2 - \sin\theta_1)^2}{(\cos\theta_1 + \cos\theta_2)^2 + (\sin\theta_2 - \sin\theta_1)^2}$

Using trigonometric identities $\sin\theta_2 - \sin\theta_1 = 2\cos\left(\frac{\theta_1+\theta_2}{2}\right)\sin\left(\frac{\theta_2-\theta_1}{2}\right)$ and $\cos\theta_1 + \cos\theta_2 = 2\cos\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\cancel{\frac{\theta_1+\theta_2}{2}} \frac{\theta_2-\theta_1}{2}\right)$, we further simplify to: $D_{min} = S \cdot \left|\sin\left(\frac{\theta_1-\theta_2}{2}\right)\right|$

10б

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3. The figure shows a graph of a cyclic process occurring with a certain amount of an ideal monatomic gas. Use the graph to determine the efficency of the heat engine operating on this cycle. Give the answer as a percentage in the form of an integer.

$$\eta = 1 - \frac{T_1}{T_2}$$

$$\frac{T_1}{T_2} = \frac{\text{area1}}{\text{area2}}$$

$$\text{area1} = \int_0^{T_1} P_{12}\, dT$$

$$\text{area2} = \int_0^{T_2} P_{12}\, dT$$

P_{12} is line passing through point 1 and 2

$$\eta = 100\% \times \left(1 - \frac{\int_0^{T_1} P_{12}\, dT}{\int_0^{T_2} P_{12}\, dT}\right) = \left(100 - \frac{100\int_0^{T_1} P_{12}\, dT}{\int_0^{T_2} P_{12}\, dT}\right)\%$$

4. Three very small positively charged balls are located at the vertices of an equilateral triangle. The first ball creates a field with a potential of φ_1 at the two remaining vertices of the triangle, the second - φ_2, and the third - φ_3. Determine the resulting force F_1 with which the second and third balls act on the first ball.

$$F_1 = \frac{\varphi_1}{k}\sqrt{\varphi_2^2+\varphi_3^2+\varphi_2\varphi_3}$$

$$\because \varphi = k\frac{q}{d} \quad \therefore q_1 = \frac{\varphi_1 d}{k} \quad q_2 = \frac{\varphi_2 d}{k} \quad q_3 = \frac{\varphi_3 d}{k}$$

$$F_{12} = k\frac{q_1 q_2}{d^2} = \frac{\varphi_1\varphi_2}{k}, \quad F_{13} = \frac{\varphi_1\varphi_3}{k}$$

$$F_1 = \sqrt{F_{12}^2+F_{13}^2+2F_{12}F_{13}\cos(60)} = \frac{\varphi_1}{k}\sqrt{\varphi_2^2+\varphi_3^2+\varphi_2\varphi_3}$$

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5. A $5000 m^3$ hot air balloon is filled with hydrogen at a ~~pe~~pressure of 110kPa. It moves of 1%. The atmospheris pressure is 100kPa, the temperature of hydrogen is equal to the ambient temperature. At ~~st~~ some point, the balloon enters an area with a humidity of 90% and begins to lower ~~i~~itself. How much weigt of the ballast must be dropped from the ballon in order to stop the descent? The saturated steam pressure of watter at 43°C is 9000Pa.

Initial conditions (10% relative humidity):

$0.1 \times 9000 Pa = 900 Pa$

$100000 Pa - 900 Pa = 99100 Pa$

$\frac{99100 \times 0.02897}{8.314 \times 316.15} \approx 1.092 kg/m^3$

$\frac{900 \times 0.018015}{8.314 \times 316.15} \approx 0.00617 kg/m^3$

Total density: $1.092 kg/m^3 + 0.00617 kg/m^3 \approx 1.098 kg/m^3$

Final conditions (90% relative humidity):

$0.9 \times 9000 Pa = 8100 Pa$

$100000 Pa - 8100 Pa = 91900 Pa$

$\frac{91900 \times 0.02897}{8.314 \times 316.15} \approx \cancel{0.0555}\ 1.013 kg/m^3$

$\frac{8100 \times 0.018015}{8.314 \times 316.15} \approx 0.0555 kg/m^3$

Total density $= 1.013 kg/m^3 + 0.0555 kg/m^3 \approx 1.0685 kg/m^3$

$1.098 kg/m^3 \times 5000 m^3 \times 9.81 m/s^2 - 1.0685 kg/m^3 \times 5000 m^3 \times 9.81 m/s^2 \approx 1447 N$

Mass to drop $\frac{1447 N}{9.81 m/s^2} \approx 147.5 kg$

$1.098 kg/m^3 - 1.0675 kg/m^3 \approx 0.030 kg/m^3$

$0.030 kg/m^3 \times 5000 m^3 = 150 kg$

влажность - ?

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