ОРМО
Открытая региональная межвузовская олимпиада

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1. Two track and field athletes, Anton(A) and Boris(B), compete in running with a distance of s. Each of them followed their own tactics: Anton ran with constant acceleration for the first third of the way, ran with constant velocity v for the second third of the way, and slowed down with the same acceleration for the last third of the way. Boris ran at constant acceleration for the first third of the time, ran at constant speed v for the second third of the time (the same as Anton's), and slowed down for the last third of the time with the same acceleration as at the start. Which of the athletes will finish first? By what time will he overtake his opponent?

Anton

First: use $v^2 = 2a \cdot \frac{s}{3}$

Second $t_{A_2} = \frac{s}{3v}$

Last: Deceleration $-a$ to rest

Time taken $t_{A_3} = 3v$. total time

$$T_A = t_{A_1} + t_{A_2} + t_{A_3} = \frac{5s}{3v}$$

Boris

First: Constant accleration a to reach speed v.

From $v = a \cdot 4$ get $a = \frac{3v}{T_B}$

$S_{B_1} = \frac{1}{2} a \left(\frac{T_B}{3}\right)^2 = \frac{vT_B}{6}$

Second $\frac{T_B}{3}$: Constant velocity v.

$S_{B_2} = \frac{vT_B}{3}$

Last: $\frac{T_B}{3}$: Deceleration $-a$ to rest

$S_{B_3} = \frac{vT_B}{6}$

$S = S_{B_1} + S_{B_2} + S_{B_3} = \frac{2vT_B}{3}$

$T_B = \frac{3s}{2v}$

Anton's total time: $T_A = \frac{5s}{3v}$

Boris's total time $= T_B = \frac{3s}{2v}$

$\Delta t = T_A - T_B = \frac{5s}{3v} - \frac{3s}{2v} = \frac{s}{6v}$

Boris finishes the faster

$v = ?$ 1Б5

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2. Two jugglers simultaneously throw balls to each other with the same initial velocity v_0, but at different angles to the horizon. Determine the minimum distance between the during the flight. The distance between the jugglers is S. Balls are thrown and caught at the same height. Ignore the air resistance.

left ball

$$\begin{cases} x_1(t) = v_0 \cos\theta_1 \cdot t \\ y_1(t) = v_0 \sin\theta_1 \cdot t - \frac{1}{2} g t^2 \end{cases}$$

Right ball

$$\begin{cases} x_2(t) = S - v_0 \cos\theta_2 \cdot t \\ y_2(t) = v_0 \sin\theta_2 \cdot t - \frac{1}{2} g t^2 \end{cases}$$

The squared distance between the two balls is:

$$D^2(t) = [S - v_0 t(\cos\theta_1 + \cos\theta_2)]^2 + [v_0 t(\sin\theta_2 - \sin\theta_1)]^2$$

Solving for t gives:

$$t = \frac{S(\cos\theta_1 + \cos\theta_2)}{v_0[(\cos\theta_1 + \cos\theta_2)^2 + (\sin\theta_2 - \sin\theta_1)^2]}$$

$$D^2_{min} = S^2 \cdot \frac{(\sin\theta_2 - \sin\theta_1)^2}{(\cos\theta_1 + \cos\theta_2)^2 + (\sin\theta_2 - \sin\theta_1)^2}$$

use $\sin\theta_2 - \sin\theta_1 = 2\cos\left(\frac{\theta_1+\theta_2}{2}\right)\sin\left(\frac{\theta_2-\theta_1}{2}\right)$ and $\cos\theta_1 + \cos\theta_2 = 2\cos\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_2-\theta_1}{2}\right)$

$$D_{min} = S \cdot \left|\sin\left(\frac{\theta_1-\theta_2}{2}\right)\right|.$$

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Место для скобы

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3. The figure shows a graph of a cyclic process occurring with a certain amount of an ideal monatomic gas. Use the graph to determine the efficiency of the heat engine operating on this cycle. Give the answer as a percentage in the form of an integer

$$D_{min} = \frac{|r_0 \times V_{rel}|}{|V_{rel}|}$$

$$D_{min} = \frac{S v_0 |\sin\theta - \cos\theta|}{v_0 \sqrt{2}} = \frac{S|\sin\theta - \cos\theta|}{\sqrt{2}}$$

$$D_{min} = \frac{S}{\sqrt{2}} \sqrt{1 - \frac{gS}{v_0^2}}$$

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4.

The potential at a distance d from a charge q is $\varphi = k\frac{q}{d}$

Charges: $q_1 = \frac{\varphi_1 d}{k}$, $q_2 \frac{\varphi_2 d}{k}$, $q_3 = \frac{\varphi_3 d}{k}$

$F_{12} = k\frac{q_1 q_2}{d^2} = \frac{\varphi_1\varphi_2}{k}$ $F_{13} = \frac{\varphi_1 \varphi_3}{k}$

$F_1 = \sqrt{F_{12}^2 + F_{13}^2 + 2F_{12}F_{13}\cos(60^\circ)} = \frac{\varphi_1}{k}\sqrt{\varphi_2^2 + \varphi_3^2 + \varphi_2\varphi_3}$

$F_2 = \frac{\varphi_2}{k}\sqrt{\varphi_1^2 + \varphi_3^2 + \varphi_1\varphi_3}$

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5. ~~Initi~~ Initial conditions

Vapor pressure: $0.1 \times 9000 Pa = 900 Pa$

Dry air pressure: $100000 Pa - 900 Pa = 99100 Pa$

Density of dry air: $\frac{99100 \times 0.02897}{8.314 \times 316.15} \approx 1.092 kg/m^3$

Density of water vapor: $\frac{900 \times 0.018015}{8.314 \times 316.15} \approx 0.00617 kg/m^3$

Total density: $1.092 + 0.00617 \approx 1.098 kg/m^3$

Final conditions

$0.9 \times 9000 Pa = 8100 Pa$

$100000 Pa - 8100 Pa = 91900 Pa$

$\frac{91900 \times 0.02897}{8.314 \times 316.15} \approx 1.013 kg/m^3$

$\frac{8100 \times 0.018015}{8.314 \times 316.15} \approx 0.0555 kg/m^3$

$1.013 + 0.0555 \approx 1.0685 kg/m^3$

Calculate the difference in buoyant force

$1.098 kg/m^3 \times 5000 m^3 \times 9.81 m/s^2 \approx 53906 N$

$1.0685 kg/m^3 \times 5000 m^3 \times 9.8 m/s^2 \approx 52459 N$

$53906 N - 52459 N \approx 1447 N$

Mass to drop: $\frac{1447 N}{9.81 m/s^2} \approx 147.5 kg$

Difference in air density: $0.030 kg/m^3$

Mass difference $0.030 kg/m^3 \times 5000 m^3 = 150 kg$

Влажность?

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