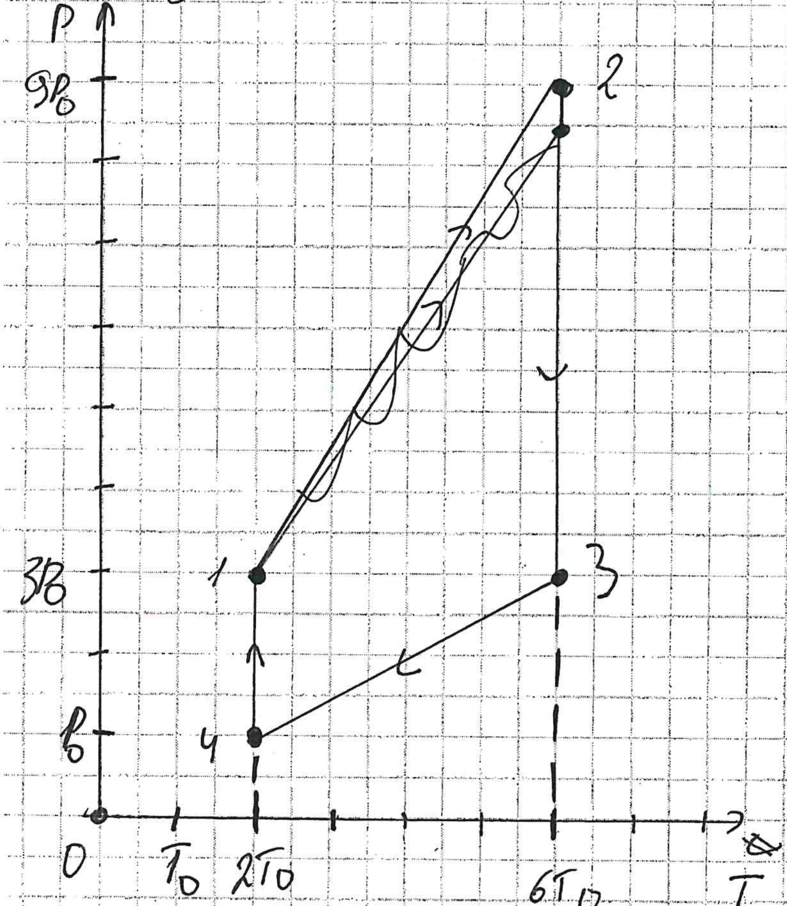
ОРМО
Открытая региональная межвузовская олимпиада

Общий балл	Дата	Ф.И.О. Жюри	Подпись
7,8б		Червинская Ю. Ю.	

Шифр 320093

Задача 3.



$$\eta = \frac{A_ц}{Q_н} = \frac{Q_н - Q_х}{Q_н}.$$

1-2: $PV = \nu RT$.

$$P = \frac{\nu R}{V} \cdot T \text{ - изохора.}$$

$A_{12} = 0$; $Q_{12} = \Delta U_{12} > 0$.

2-3 - изотерма.

$Q_{23} = A_{23} \cancel{\neq} > 0$.

34 - изохора.

$Q_{34} < 0$.

41 - изотерма, $Q_{41} \cancel{\neq} < 0$.

$$Q_{12} = \Delta U_{12} = \frac{3}{2}\nu R(T_2 - T_1) = \frac{3}{2}\nu R(6T_0 - 2T_0) = \frac{3\nu R \cdot \cancel{4}^{2}T_0}{\cancel{2}} = 6\nu RT_0$$

~~$Q_{41} = \Delta U$~~ $Q_{34} = \frac{3}{2}\nu R(6T_0 - 2T_0) = 6\nu RT_0$.

~~$Q_{23} = A_{23} = \int_{V_2}^{V_3} pdV$~~

$$Q_{23} = A_{23} = \nu RT_2 \cdot \ln\frac{V_3}{V_2};\quad P_3V_3 = \nu RT_3 \Rightarrow 3P_0 \cdot V_3 = \nu R \cdot 6T_0$$

$$V_3 = \frac{2\nu RT_0}{P_0}$$

$$P_2V_2 = \nu RT_2 \Rightarrow 9P_0V_2 = \nu R \cdot 6T_0$$

$$V_2 = \frac{\nu R \cdot 6T_0}{9P_0} = \frac{\nu RT_0 \cdot 2}{3}$$

$$A_{23} = \nu R \cdot 6T_0 \cdot \ln \cdot \frac{2 \cdot 3}{2} = \nu R \cdot 6T_0 \cdot \ln 3 \approx \nu R \cdot 6T_0 \cdot 1{,}099 = \nu RT_0 \cdot 6{,}594$$

~~$A_{41} = \nu R T_4 \cdot \ln\frac{V_4}{V_{21}} = \nu R \cdot 2T_0 \cdot \ln\frac{V_4}{V_2}$~~,

$A_{41} = \nu R T_4 \cdot \ln\frac{V_1}{V_4} = 2\nu R T_0 \cdot \ln\frac{V_1}{V_4}$; $V_1 \cdot 3P_0 = \nu R \cdot 2T_0$

$A_{41} = 2\nu R T_0 \cdot \ln\frac{2}{3\cdot 2} = 2\nu R T_0 \cdot \ln\frac{1}{3} =$ $\quad V_1 = \frac{2\nu R T_0}{3P_0}$

$= 2\nu R T_0 \cdot (-1{,}099) =$ $\quad V_4 \cdot P_0 = 2\nu R T_0 \Rightarrow V_4 = \frac{2\nu R T_0}{P_0}$

$= -2{,}198\,\nu R T_0$.

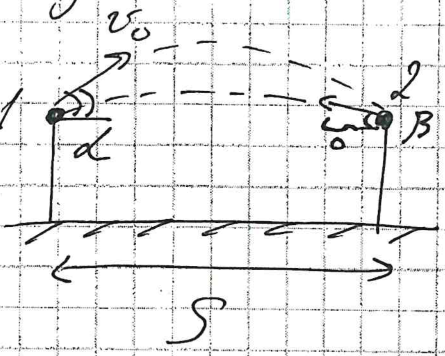
$\eta = \frac{A_ц}{Q_н}$; $A_ц = A_{23} + A_{41} = \nu R T_0(6{,}594 - 2{,}198) =$
$= \nu R T_0 \cdot 4{,}396$.

$Q_н = Q_{12} + Q_{23} = \Delta U_{12} + A_{41}$ ~~$= 6\nu R T_0 + -2{,}198\,\nu R T_0 =$~~
$= 6\nu R T_0 + 6{,}594\,\nu R T_0 = \nu R T_0 \cdot 12{,}59$

$\eta = \frac{A_ц}{Q_н} = \frac{\nu R T_0 \cdot 4{,}396}{\nu R T_0 \cdot 12{,}59} \cdot 100\% = 0{,}3492 \cdot 100\% \approx 35\%$

Ответ: 35%   20б

Задача 2.

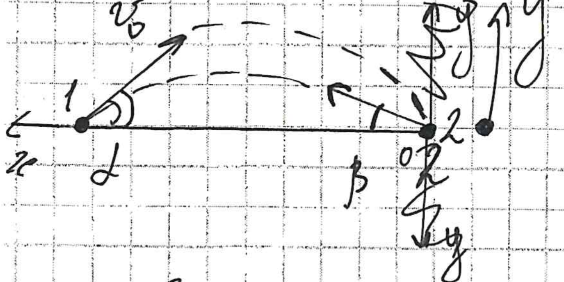


$y_1 = v_0 \sin\alpha \cdot t - \frac{gt^2}{2}$

$y_2 = v_0 \sin\beta \cdot t - \frac{gt^2}{2}$

$\Delta y = y_1 - y_2 = v_0(\sin\alpha - \sin\beta)t$.

~~$x_1 = v_0\cos\alpha \cdot t$~~
~~$x_2 = v_0 \cdot \sin\alpha \cdot t$~~ ~~$\Rightarrow x_1 - x_2 = v_0(\cos\alpha - \cos\beta)t = \Delta x$~~.



$x_1 = S - v_0\cos\alpha \cdot t$
$x_2 = v_0 \cos\beta \cdot t$ $\quad \Rightarrow \Delta x = x_1 - x_2 =$
$= S - v_0 t(\cos\alpha + \cos\beta)$.

$\Delta S^2 = \Delta y^2 + \Delta x^2 = v_0^2(\sin\alpha + \sin\beta)^2 t^2 + (S - v_0 t(\cos\alpha + \cos\beta))^2 =$
$= (v_0(\sin\alpha + \sin\beta)t)^2 + S^2 - 2Sv_0 t(\cos\alpha + \cos\beta) + v_0^2 t^2(\cos\alpha + \cos\beta)^2 =$

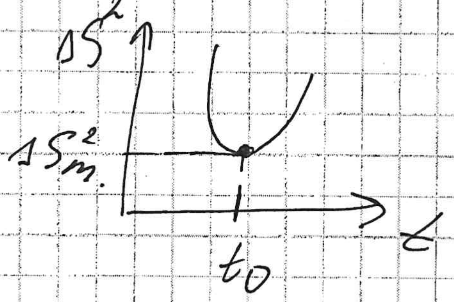
$= v_0^2 t^2 ((\sin\alpha+\sin\beta)^2+(\cos\alpha+\cos\beta)^2) + S^2 - 2Sv_0t(\cos\alpha+\cos\beta) =$

$\cancel{(\sin^2\alpha+\sin}$ $(\sin\alpha+\sin\beta)^2+(\cos\alpha+\cos\beta)^2 = \sin^2\alpha + 2\sin\alpha\sin\beta +$

$+ \cancel{\cos\alpha}\cdot\sin^2\beta + \cos^2\alpha + 2\cos\alpha\cdot\cos\beta + \cos^2\beta = 1+1+2(\sin\alpha\sin\beta +$

$+\cos\alpha\cos\beta) = 2(1+\sin\alpha\sin\beta+\cos\alpha\cdot\cos\beta) = 2(1+\cos(\alpha-\beta))$

$$\Delta S^2 = v_0^2t^2(2(1+\cos(\alpha-\beta))) + S^2 - 2Sv_0t(\cos\alpha+\cos\beta)$$



$$t_0 = \frac{-b}{2a} = \frac{\cancel{2}Sv_0(\cos\alpha+\cos\beta)}{\cancel{4}_2\,\cancel{2}v_0^2(1+\cos(\alpha-\beta))} = \frac{S(\cos\alpha+\cos\beta)}{2v_0(1+\cos(\alpha-\beta))}$$

$$\Delta S_m^2 = 2v_0^2(1+\cos(\alpha-\beta))\cdot\frac{S^2(\cos\alpha+\cos\beta)^2}{v_0^2(1+\cos(\alpha-\beta))^2} + S^2 -$$

$$-2Sv_0\cdot(\cos\alpha+\cos\beta)\cdot\frac{S(\cos\alpha+\cos\beta)}{v_0(1+\cos(\alpha-\beta))} =$$

$$= S^2 + 2S^2\cdot\frac{(\cos\alpha+\cos\beta)^2}{1+\cos(\alpha-\beta)}$$

$$\Delta S_m^2 = \cancel{2v_0^2}(\cancel{1+\cos(\alpha-\beta)})\cdot\frac{S^2(\cos\alpha+\cos\beta)^2}{2\,\cancel{4v_0^2}(1+\cos(\alpha-\beta))^{\cancel{2}}} + S^2 -$$

$$-\cancel{2}S\cancel{v_0}(\cos\alpha+\cos\beta)\cdot\frac{S(\cos\alpha+\cos\beta)}{\cancel{2v_0}(1+\cos(\alpha-\beta))} = S^2 + \frac{S^2}{2}\cdot\frac{(\cos\alpha+\cos\beta)^2}{(1+\cos(\alpha-\beta))}$$

$$- S^2\cdot\frac{(\cos\alpha+\cos\beta)^2}{1+\cos(\alpha-\beta)} = S^2 - \frac{S^2}{2}\cdot\frac{(\cos\alpha+\cos\beta)^2}{1+\cos(\alpha-\beta)}$$

Расстояние минимально тогда, когда ~~минимально~~ максимально выражение $\frac{(\cos\alpha+\cos\beta)^2}{1+\cos(\alpha-\beta)}$.

~~$1+\cos(\alpha-\beta)$ должно быть мин. $\Rightarrow \cos(\alpha-\beta)=0$~~

Общий балл	Дата	Ф.И.О. Жюри	Подпись

ОРМО
Открытая региональная межвузовская олимпиада

Шифр

$\cancel{\alpha-\beta=\frac{\pi}{2}} \Rightarrow$ Мячики летят на одной высоте $\Rightarrow$

$y_1 = y_2 \Rightarrow v_0 \sin\alpha \cdot t_1 = v_0 \sin\beta \cdot t_2$, где t_1 и t_2 — время полета 1-го и второго

$\cancel{S=S}$

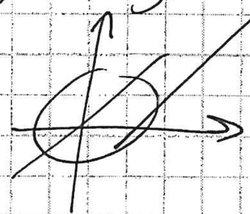
$\left.\begin{aligned} 0 &= S - v_0\cos\alpha \cdot t_1 \\ S &= v_0 \cos\beta \cdot t_2 \end{aligned}\right\} \Rightarrow \cos\alpha \cdot t_1 = \cos\beta \cdot t_2$

$t_2 = \frac{\cos\alpha}{\cos\beta} \cdot t_1$

$\sin\alpha \cdot \cancel{t_1} = \sin\beta \cdot \frac{\cos\alpha}{\cos\beta} \cdot \cancel{t_1} \;|: \cos\alpha$

$\operatorname{tg}\alpha = \operatorname{tg}\beta \Rightarrow \cancel{\alpha = \pi + \beta}$.

$0 \le \alpha \le 90°$; $0 \le \beta \le 90°$.



$y_1 = y_2 \Rightarrow v_0 \sin\alpha \cdot t_1 - \frac{g t_1^2}{2} = v_0 \sin\beta\, t_2 - \frac{g t_2^2}{2}$

$S = v_0 \cos\alpha \cdot t_1 = v_0 \cos\beta\, t_2$

$t_2 = \frac{\cos\alpha}{\cos\beta} \cdot t_1$

$v_0 \sin\alpha \cdot \cancel{t_1} - \frac{g t_1^{\cancel{2}}}{2} = v_0 \sin\beta \cdot \frac{\cos\alpha}{\cos\beta} \cancel{t_1} - \frac{g}{2} \cdot t_1^{\cancel{2}} \cdot \frac{\cos^2\alpha}{\cos^2\beta}$

$v_0 \sin\alpha - \frac{g t_1}{2} = v_0 \sin\beta \cdot \frac{\cos\alpha}{\cos\beta} - \frac{g t_1}{2} \cdot \frac{\cos^2\alpha}{\cos^2\beta}$

$\Delta S_m^2 = S^2 - \frac{S^2}{2} = \frac{S^2}{2}$

$S_m = \frac{S}{\sqrt{2}}$.

Ответ: $\frac{S}{\sqrt{2}}$ — 1,25

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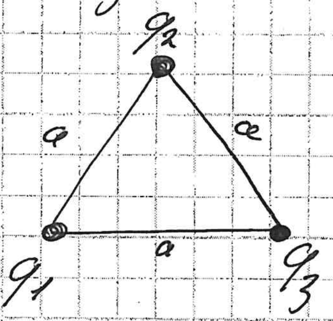
Общий балл	Дата	Ф.И.О. Жюри	Подпись

Шифр

Задача 4.



$\varphi_1 = \frac{kq_1}{a} \Rightarrow q_1 = \frac{\varphi_1 a}{k}$

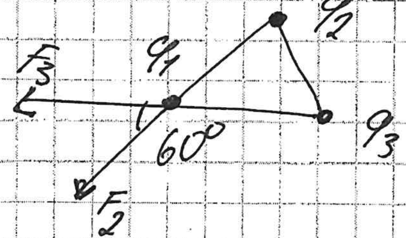
$\varphi_2 = \frac{kq_2}{a} \Rightarrow q_2 = \frac{\varphi_2 a}{k}$

$\varphi_3 = \frac{kq_3}{a} \Rightarrow q_3 = \frac{\varphi_3 a}{k}$

$F_2 = \frac{kq_1q_2}{a^2} =$

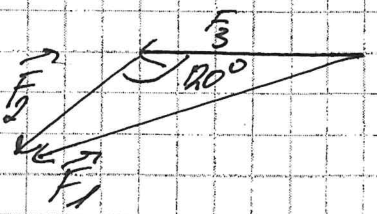
$= \frac{k}{a^2}\cdot\frac{a}{k}\cdot\varphi_1\cdot\frac{a}{k}\cdot\varphi_2 =$

$= \frac{\varphi_1\varphi_2}{k}$



$\vec{F_1} = \vec{F_3} + \vec{F_2}$

$F_3 = \frac{kq_1q_3}{a^2} = \frac{\varphi_1\varphi_3}{k}$



$F_1^2 = F_2^2 + F_3^2 - 2F_2F_3\cdot\cos 120° =$

$= F_2^2 + F_3^2 + F_2F_3 = \frac{1}{k^2}\left((\varphi_1\varphi_2)^2 + (\varphi_1\varphi_3)^2 + \varphi_1\varphi_2\cdot\varphi_1\varphi_3\right) =$

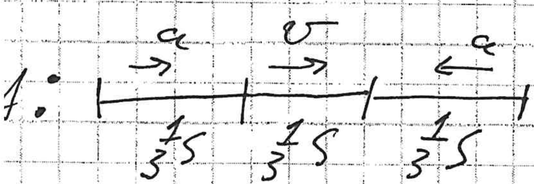
$= \frac{\varphi_1^2}{k^2}\left(\varphi_2^2 + \varphi_3^2 + \varphi_2\varphi_3\right)$

$F_1 = \frac{\varphi_1}{k}\sqrt{\varphi_2^2 + \varphi_3^2 + \varphi_2\varphi_3}$

Ответ: $F_1 = \frac{\varphi_1}{k}\sqrt{\varphi_2^2 + \varphi_3^2 + \varphi_2\varphi_3}$ = ?

18б

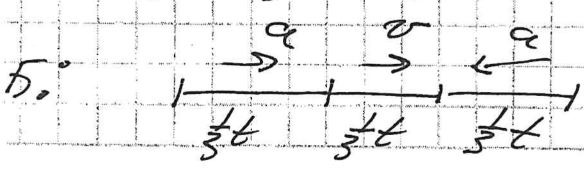
Задача 1.



A: $\frac{1}{3}S = \frac{at_1^2}{2} \Rightarrow at_1^2 = \frac{2}{3}S$

$t_1 = \sqrt{\frac{2}{3}\cdot\frac{S}{a}}$

$\frac{1}{3}S = v\tau \Rightarrow \tau = \frac{1S}{3v}$.



$\frac{1}{3}S = vt_2 - \frac{at_2^2}{2}$; $v = at_2$; $t_2 = t_1$

$t_2 = \frac{v}{a}$

Общий балл	Дата	Ф.И.О. Жюри	Подпись

Шифр

$T_1 = t_1 + t_2 + \tau = 2t_1 + \tau = 2\sqrt{\frac{2}{3}\cdot\frac{S}{a}} + \frac{S}{3v} = \frac{2v}{a} + \frac{S}{3v}$

Б: ~~x~~ $v = \frac{1}{3}T_2 \cdot a'$.

$x_1 = \frac{a't^2}{2} = \frac{a'\cdot\frac{1}{9}T_2^2}{2} = \frac{a'T_2^2}{18}$

$x_2 = \frac{T_2}{3}v$

$x_3 = v\cdot\frac{T_2}{3} - \frac{a'T_2^2}{18}$

$x_1 + x_2 + x_3 = S \Rightarrow \frac{a'T_2^2}{18} + \frac{T_2 v}{3} + \frac{T_2 v}{3} - \frac{a'T_2^2}{18} = \frac{2T_2 v}{3} = S$

$T_2 = \frac{3S}{2v}$.

к А: $t_1 = \frac{v}{a}$; $t_1 = \sqrt{\frac{2S}{3a}} \Rightarrow \frac{v^2}{a^{\cancel{2}}} = \frac{2S}{3\cancel{a}}$

$a = \frac{3v^2}{2S}$

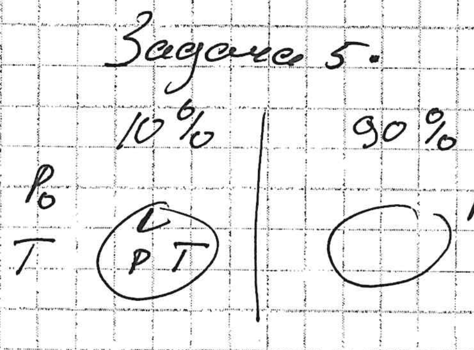
Тогда $T_1 = \frac{2v\cdot 2S}{3v^{\cancel{2}}} + \frac{S}{3v} = \frac{4S}{3v} + \frac{S}{3v} = \frac{5S}{3v}$

А прибежит позже Б на $\Delta T = T_1 - T_2 = \frac{5S}{3v} - \frac{3S}{2v} =$

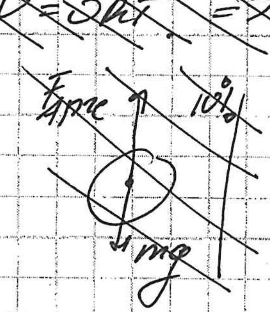
$= \frac{10S}{6v} - \frac{9S}{6v} = \frac{S}{6v}$.

Ответ: $\Delta t = \frac{S}{6v}$, Борис прибежит первым. 20б

Задача 5.



~~$PV = \nu RT \Rightarrow \nu = \frac{PV}{RT} =$~~



$\varphi = \frac{P_п}{P_{н.п.}} \Rightarrow P_п = \varphi\cdot P_{н.п.}$

$P_{п1} = \varphi_1\cdot P_{н.п} = 0{,}1\cdot 9000 =$
$= 900$ Па.

?

~~$P_0 = P_{с.в} + P_{п1} \Rightarrow P_{с.в} = P_0 - P_{п.1} = 100\,000 - 900 =$~~

$P_{в.1} = P_0 + P_{п1} = 100\,900$ Па — давл. воздуха при $\varphi_1 = 10\%$.

$P_{п.2} = \varphi_2 \cdot P_{н.п} = 0{,}9\cdot 9000 = 8100$ Па.

Общий балл	Дата	Ф.И.О. Жюри	Подпись

Шифр

$\Delta P = P_{n.2} - P_{n.1} = 8100 - 900 = 7200$ Па.

$pV = const.$

$\cancel{P_1 V_1 = P_2 V_2 \Rightarrow V_2 = \frac{P_1}{P_2} V_1.}$

$\cancel{\Delta V = V_1 - V_2 = V\left(1 - \frac{P_1}{P_2}\right) = V \cdot \left(1 - \frac{900}{8100}\right) = V \cdot 0{,}989}$

$\Delta P_0 V = \frac{\Delta m}{\mu} RT \Rightarrow \Delta m = \frac{\mu \Delta P_0 V}{RT} = \frac{2 \cdot 10^{-3} \cdot 7200 \cdot 5 \cdot 10^{3}}{8{,}31 \cdot 316} =$

$= \frac{10 \cdot 7200}{2626} = 27{,}42$ (кг).

Ответ: 27,42 (кг)

8б