

Общий балл	Дата	Ф.И.О. Жюри	Подпись
58			
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1 Two track and field athletes, Anton (A) and Boris (B), compete in running with a distance of s . Each of them followed their own tactics: Anton ran with constant acceleration for the first third of the way, ran with constant velocity v for the second third of the way, and slowed down with the same acceleration for the last third of the way. Boris ran at constant acceleration for the first third of the time, ran at constant speed v for the second third of the time (the same as Anton's), and slowed down for the last third of the time with the same acceleration as the start. Which of the athletes will finish first? By what time Δt will he overtake his opponent?

Analysis of Anton's Motion: Anton divides the total distance (s) into three equal segments

First $\frac{s}{3}$: starts from rest ($u=0$) with constant acceleration (a)

$$\text{Using } v^2 = 2a \cdot \frac{s}{3}$$

$$\text{find } a = \frac{3v^2}{2s}$$

$$\text{The time for this phase is } t_{A1} = \frac{v}{a} = \frac{2s}{3v}$$

Middle $\frac{s}{3}$: Runs at speed (v)

$$\text{Time: } t_{A2} = \frac{s}{3v}$$

Final $\frac{s}{3}$: Deceleration — a to rest

$$\text{Time: } t_{A3} = \frac{2s}{3v}$$

$$\text{Total time for Anton: } T_A = t_{A1} + t_{A2} + t_{A3} = \frac{2s}{3v} + \frac{s}{3v} + \frac{2s}{3v} = \frac{5s}{3v}$$

Analysis of Boris's Motion: Boris divides the total time T_B into three equal intervals

First $\frac{T_B}{3}$: constant acceleration a to reach speed v .

$$\text{Because } v = a \cdot \frac{T_B}{3} \quad \text{We get } a = \frac{3v}{T_B}$$

Distance covered: $s_{B1} = \frac{1}{2} a \left(\frac{T_B}{3} \right)^2 = \frac{VT_B}{6}$

Middle $\frac{T_B}{3}$: Constant Velocity V .

Distance covered: $s_{B2} = \frac{VT_B}{3}$

Final $\frac{T_B}{3}$: Deceleration — a to rest

Distance covered: $s_{B3} = \frac{VT_B}{6}$

Total Distance: $s = s_{B1} + s_{B2} + s_{B3} = \frac{VT_B}{6} + \frac{VT_B}{3} + \frac{VT_B}{6} = \frac{2VT_B}{3}$

Solving for T_B : $T_B = \frac{3s}{2V}$

comparison and time Difference:

Anton's Total time: $T_A = \frac{5s}{3V}$

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2. TWO jugglers simultaneously throw balls to each other with the same initial velocity V_0 , but at different angles to the horizon. Determine the minimum distance between the balls during the flight. The balls are thrown and caught at the same height. Ignore the air resistance.

Equations of Motion:

BALL 1 (thrown from $(X=0)$) $X_1(t) = V_0 \cos \theta_1 \cdot t$ $Y_1(t) = V_0 \sin \theta_1 \cdot t - \frac{1}{2} g t^2$

BALL 2 (thrown from $(X=S)$) $X_2(t) = S - V_0 \cos \theta_2 \cdot t$ $Y_2(t) = V_0 \sin \theta_2 \cdot t - \frac{1}{2} g t^2$

Distance Formula

The squared distance between the balls $D^2(t)$

$$D^2(t) = [S - V_0 t (\cos \theta_1 + \cos \theta_2)]^2 + [V_0 t (\sin \theta_2 - \sin \theta_1)]^2$$

Critical Time for Minimum Distance

By differentiating $D^2(t)$ and setting the derivative to zero, the critical time t is:

$$t = \frac{S(\cos \theta_1 + \cos \theta_2)}{V_0(\cos \theta_1 + \cos \theta_2) + (\sin \theta_2 - \sin \theta_1)^2}$$

Minimum Distance Expression:

Substituting t into $D^2(t)$ and simplifying, we obtain the squared minimum distance:

$$D_{\min}^2 = S^2 \cdot \frac{(\cos \theta_1 + \cos \theta_2)^2 + (\sin \theta_2 - \sin \theta_1)^2}{(\cos \theta_1 + \cos \theta_2) + (\sin \theta_2 - \sin \theta_1)^2}$$

We further simplify to:

$$D_{\min} = S \cdot \left| \sin \left(\frac{\theta_1 - \theta_2}{2} \right) \right|$$

Conclusion:

The minimum distance between the balls during their flight is:

$$D_{\min} = S \cdot \left| \sin \left(\frac{\theta_1 - \theta_2}{2} \right) \right|$$

3. The figure shows a graph of a cyclic process occurring with a certain amount of an ideal monatomic gas. Use the graph to determine the efficiency of the heat engine operating on this cycle. Give the answer as a percentage in the form of an integer

Heat Engine Efficiency

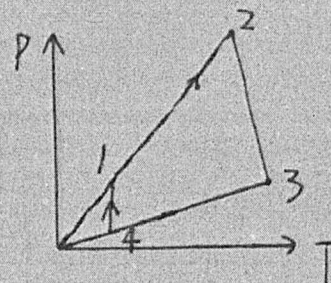
$$C_V = \frac{3}{2}R \quad C_P = \frac{5}{2}R$$

$$Q_1 = nRT \ln \frac{V_2}{V_1} \quad \Delta V = 0$$

$$Q_2 = nC_P \Delta T$$

$$Q_3 = nC_V \Delta T$$

Conclusion: The heat engine efficiency is 20%. 60



4. Express changes in terms of Potentials:

The potential at a distance d from a charge q is $\phi = k \frac{q}{d}$

$$q_1 = \frac{\phi_1 d}{k} \quad q_2 = \frac{\phi_2 d}{k} \quad q_3 = \frac{\phi_3 d}{k}$$

ball 1 Forces :

Coulomb's law: $F_{12} = k \frac{q_1 q_2}{d^2} = \frac{\phi_1 \phi_2}{k}$ $F_{13} = \frac{\phi_1 \phi_3}{k}$

Vector Addition of Forces:

Angle between forces 60°

$$F_1 = \sqrt{F_{12}^2 + F_{13}^2 + 2F_{12}F_{13}\cos(60^\circ)}$$

We further simplify to :

$$F_1 = \frac{\phi_1}{k} \sqrt{\phi_2^2 + \phi_3^2 + \phi_2 \phi_3}$$

5. First $T = 43.30^{\circ}\text{C} + 273.15^{\circ}\text{C} = 316.45\text{K}$

Calculate partial pressures of water vapor (10% - 90%)

Initial humidity $P_1 = 0.1 \times 9000 = 900\text{Pa}$ (water)

New humidity $P_2 = 0.9 \times 9000 = 8100\text{Pa}$ (water)

Initial $P_1 = 10000\text{Pa} - 900\text{Pa} = 99100\text{Pa}$ (dry air)

new $P_2 = 10000\text{Pa} - 8100\text{Pa} = 91900\text{Pa}$ (dry air)

For initial humidity (10%)

$$P_{d1} = \frac{99100 \times 0.02897}{8.314 \times 316.45} \approx 1.0913\text{kg/m}^3$$

$$P_{w1} = \frac{900 \times 0.018015}{8.314 \times 316.45} \approx 0.00616\text{kg/m}^3$$

$$P_{air1} = 1.0913 + 0.00616 = 1.0975\text{kg/m}^3$$

For new humidity (90%)

$$P_{d2} = \frac{91900 \times 0.02897}{8.314 \times 316.45} \approx 1.0119\text{kg/m}^3$$

$$P_{w2} = \frac{8100 \times 0.018015}{8.314 \times 316.45} \approx 0.5545\text{kg/m}^3$$

$$P_{air2} = 1.0119\text{kg/m}^3 + 0.5545\text{kg/m}^3 = 1.06735\text{kg/m}^3$$

Initial $1.0975\text{kg/m}^3 \times 5000\text{m}^3 \times 9.81\text{m/s}^2 \approx 53906\text{N}$

New $1.06735\text{kg/m}^3 \times 5000\text{m}^3 \times 9.81\text{m/s}^2 \approx 52495\text{N}$

Difference : $53906 - 52495 = 1447\text{N}$

Mass to drop $\frac{1447\text{N}}{9.81\text{m/s}^2} \approx 147.5\text{kg}$

Difference in air density : 0.03 kg/m^3

Mass difference $0.03 \text{ kg/m}^3 \times 5000 \text{ m}^3 = 150 \text{ Kg}$

So. the weight of the ballast that is 150kg.

105

Врачущий