

Общий балл	Дата	Ф.И.О. Жюри	Подпись
77			<i>Смирнов</i>
Шифр			041912

1. Two track and field athletes, Anton (A) and Boris (B), compete in running with a distance of S . Each of them followed their own tactics: Anton ran with constant acceleration for the first of the way, ran with constant velocity v for the second third of the way. Boris ran at constant acceleration for the first third of the time, ran at constant speed v for the second third of the time (the same as Anton's), and slowed down for the last third of the time with the same acceleration as at the start. Which of the athletes will finish first? By what time Δt will he overtake his opponent?

Anton's Strategy

First $\frac{S}{3}$: Constant acceleration α , starting from rest ($v_0 = 0$) to reach speed v .

Using $v^2 = 2\alpha \cdot \frac{S}{3}$, we find $\alpha = \frac{3v^2}{2S}$.

Time taken: $t_{A1} = \frac{v}{\alpha} = \frac{2S}{3v}$

Middle $\frac{S}{3}$: Time taken: $t_{A2} = \frac{v}{v} = \frac{2S}{6v}$

Final $\frac{S}{3}$: Deceleration - to rest

Time taken: $t_{A3} = \frac{2S}{3v}$

Total Time: $T_A = t_{A1} + t_{A2} + t_{A3} = \frac{2S}{3v} + \frac{S}{3v} + \frac{2S}{3v} = \frac{5S}{3v}$

Boris's Strategy

First $\frac{T_B}{3}$: $v = \alpha \cdot \frac{T_B}{3}$, get $\frac{3v}{T_B}$

$S_{B1} = \frac{1}{2} \alpha \left(\frac{T_B}{3}\right)^2 = \frac{vT_B}{6}$

Middle $\frac{T_B}{3}$: $S_{B2} = \frac{vT_B}{3}$

Final $\frac{T_B}{3}$: $S_{B3} = \frac{1}{2} (-\alpha) \left(\frac{T_B}{3}\right)^2 = \frac{vT_B}{6}$

$S = S_{B1} + S_{B2} + S_{B3} = \frac{2vT_B}{3}$ $T_B = \frac{3S}{2v}$

$\Delta t = T_A - T_B = \frac{5S}{3v} - \frac{3S}{2v} = \frac{S}{6v}$

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2. Equations of Motion

Left Ball (θ_1)

Position as a function of time t :

$$\begin{cases} x_1(t) = v \cos \theta_1 \cdot t, \\ y_1(t) = v_0 \sin \theta_1 \cdot t - \frac{1}{2} g t^2 \end{cases}$$

Right Ball (θ_2)

$$\begin{cases} x_2(t) = S - v_0 \cos \theta_2 \cdot t \\ y_2(t) = v_0 \sin \theta_2 \cdot t - \frac{1}{2} g t^2 \end{cases}$$

Distance Formula

$$D^2(t) = [S - v_0 t (\cos \theta_1 + \cos \theta_2)]^2 + [v_0 t (\sin \theta_2 - \sin \theta_1)]^2.$$

To find the minimum distance

$$t = \frac{S (\cos \theta_1 + \cos \theta_2)}{v_0 [(\cos \theta_1 + \cos \theta_2)^2 + (\sin \theta_2 - \sin \theta_1)^2]}$$

Substituting t into $D^2(t)$ and simplifying, we obtain the squared minimum distance

$$D_{\min}^2 = S^2 \cdot \frac{(\sin \theta_2 - \sin \theta_1)^2}{(\cos \theta_1 + \cos \theta_2)^2 + (\sin \theta_2 - \sin \theta_1)^2}$$

$$\sin \theta_2 - \sin \theta_1 = 2 \cos \left(\frac{\theta_1 + \theta_2}{2} \right) \sin \left(\frac{\theta_2 - \theta_1}{2} \right) \text{ and } \cos \theta_1 + \cos \theta_2 = 2 \cos \left(\frac{\theta_1 + \theta_2}{2} \right) \cos \left(\frac{\theta_2 - \theta_1}{2} \right),$$

we further simplify to:

$$D_{\min} = S \cdot \left| \sin \left(\frac{\theta_1 - \theta_2}{2} \right) \right|$$

Conclusion

$$|D_{\min}| = S \cdot \left| \sin \left(\frac{\theta_1 - \theta_2}{2} \right) \right|$$

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3. The figure shows a graph of a cyclic process occurring with a certain amount of an ideal monatomic gas. Use the graph to determine the efficiency of the heat engine operating on this cycle. Give the answer as a percentage in the form of a integer.

The first law of thermodynamics is $\Delta U = Q + W$

The efficiency formula of a heat engine is $\eta = \frac{W_{\text{net}}}{Q_{\text{in}}}$,

the internal energy $U = \frac{3}{2} nRT$

In an isochoric (constant-volume) process, $W = p\Delta V$, $nR\Delta T$, then $Q_p = \Delta U + W = \frac{3}{2} nR\Delta T + nR\Delta T = \frac{5}{2} nR\Delta T$

For an isochoric heating process, $Q_p = \frac{5}{2} nR\Delta T$, and then $Q_{\text{in}} = Q_v + Q_p$

We use $p = \frac{nRT}{V}$ to convert the $T-V$

Substitute W_{net} and Q_{in} into the efficiency formula $\eta = \frac{W_{\text{net}}}{Q_{\text{in}}}$

For the process from state 1 to state 2 ($W_1 = 0$), $Q_{1-2} = \frac{3}{2} nR(T_2 - T_1)$

For the process from state 2 to state 3, $W_2 = p_2(V_3 - V_2) = nR(T_3 - T_2)$, $Q_{2-3} = \frac{5}{2} nR(T_3 - T_2)$.

For the process from state 3 to state 4 ($W_3 = 0$), $Q_{3-4} = -\frac{3}{2} nR(T_3 - T_4)$

For the process from state 4 to state 1, $W_4 = p_4(V_1 - V_4) = nR(T_1 - T_4)$, $Q_{4-1} = -\frac{5}{2} nR(T_1 - T_4)$ (heat-releasing)

So, $Q_{\text{in}} = \frac{3}{2} nR(T_2 - T_1) + \frac{5}{2} nR(T_3 - T_2)$

The net work done $W_{\text{net}} = p_2(V_3 - V_2) + p_4(V_1 - V_4)$

~~So, $Q_{\text{in}} = \frac{3}{2} nR(T_2 - T_1) + \frac{5}{2} nR(T_3 - T_2)$~~ $\eta = \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{p_2(V_3 - V_2) + p_4(V_1 - V_4)}{\frac{3}{2} nR(T_2 - T_1) + \frac{5}{2} nR(T_3 - T_2)}$

$T_1 = T_4$, $V_1 = V_2$, $V_3 = V_4$, $p_2 = p_3$, $p_1 = p_4$

We get $\eta = 25\%$

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4. The expression for obtaining force is:

$$F_1 = \frac{\phi_1}{k} \sqrt{\phi_2^2 + \phi_3^2 + \phi_2 \phi_3}$$

Step-by-Step Explanation:

1. The potential at a distance d from a charge q is $\phi = k \frac{q}{d}$.

Charges: $q_1 = \frac{\phi_1 d}{k}$, $q_2 = \frac{\phi_2 d}{k}$, $q_3 = \frac{\phi_3 d}{k}$.

2. Coulomb's law: $F_{12} = k \frac{q_1 q_2}{d^2} = \frac{\phi_1 \phi_2}{k}$, $F_{13} = \frac{\phi_1 \phi_3}{k}$

3. Angle between forces: 60° :

Resultant force magnitude, $F_1 = \sqrt{F_{12}^2 + F_{13}^2 + 2F_{12}F_{13}\cos(60^\circ)} = \frac{\phi_1}{k} \sqrt{\phi_2^2 + \phi_3^2 + \phi_2 \phi_3}$

Final Answer: $F_1 = \frac{\phi_1}{k} \sqrt{\phi_2^2 + \phi_3^2 + \phi_2 \phi_3}$

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2. (1) Initial conditions (10% relative humidity):

Vapor pressure: $0.1 \times 9000 \text{ Pa} = 900 \text{ Pa}$

Dry air pressure: $100000 \text{ Pa} - 900 \text{ Pa} = 99100 \text{ Pa}$

Density of dry air: $\frac{99100 \times 0.02897}{8.314 \times 316.15} \approx 1.092 \text{ kg/m}^3$

Density of water vapor: $\frac{900 \times 0.01802}{8.314 \times 316.15} \approx 0.00617 \text{ kg/m}^3$

Total density: $1.092 + 0.00617 \approx 1.098 \text{ kg/m}^3$

Final conditions:

Vapor pressure: $0.9 \times 9000 \text{ Pa} = 8100 \text{ Pa}$

Dry air pressure: $100000 \text{ Pa} - 8100 \text{ Pa} = 91900 \text{ Pa}$

Density of dry air: $\frac{91900 \times 0.02897}{8.314 \times 316.15} \approx 0.955 \text{ kg/m}^3$

Total density: $1.013 + 0.955 \approx 1.0685 \text{ kg/m}^3$

(2) Initial buoyant force: $1.098 \text{ kg/m}^3 \times 5000 \text{ m}^3 \times 9.81 \text{ m/s}^2 \approx 53906 \text{ N}$

Final buoyant force: $1.0685 \text{ kg/m}^3 \times 5000 \text{ m}^3 \times 9.81 \text{ m/s}^2 \approx 52459 \text{ N}$

Difference in buoyant force: $53906 \text{ N} - 52459 \text{ N} \approx 1447 \text{ N}$

(3) Mass to drop: $\frac{1447 \text{ N}}{9.81 \text{ m/s}^2} \approx 147.5 \text{ kg}$

(4) Difference in air density: 0.030 kg/m^3

Mass difference: $0.030 \text{ kg/m}^3 \times 5000 \text{ m}^3 = 150 \text{ kg}$

Важно?

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