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1. Anton's Strategy:

First third of distance ($\frac{s}{3}$):

Constant acceleration a, starting from rest.

$v^2 = 2a \cdot \frac{s}{3} \longrightarrow a = \frac{3v^2}{2s}$

Time: $t_1 = \frac{v}{3v/2s} = \frac{2s}{3v}$

Second third of distance ($\frac{s}{3}$):

Constant velocity v

Time: $t_2 = \frac{s/3}{v} = \frac{s}{3v}$

Last third of distance ($\frac{s}{3}$)

Deceleration $-a$ to rest

Time: $t_3 = \frac{v}{3v/2s} = \frac{2s}{3v}$

Total Time for Anton:

$T_A = t_1 + t_2 + t_3 = \frac{2s}{3v} + \frac{s}{3v} + \frac{2s}{3v} = \frac{5s}{3v}$

Boris's Strategy:

First third of time ($\frac{T}{3}$):

Constant acceleration a.

$v = a \cdot \frac{T}{3} \rightarrow a = \frac{3v}{T}$

Distance: $s_1 = \frac{1}{2}a(\frac{T}{3})^2 = \frac{vT}{6}$

Second third of time ($\frac{T}{3}$):

Constant velocity v

Distance: $s_2 = v \cdot \frac{T}{3} = \frac{vT}{3}$

Место для скобы

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Last third of time ($\frac{T}{3}$):

Deceleration $-a$ to rest

$0 = v - a \cdot \frac{T}{3} \rightarrow a = \frac{3v}{T}$

Distance: $S_3 = v \cdot \frac{T}{3} - \frac{1}{2}a(\frac{T}{3})^2 = \frac{vT}{6}$

Total Distance for Boris

$S = S_1 + S_2 + S_3 = \frac{vT}{6} + \frac{vT}{3} + \frac{vT}{6} = \frac{vT}{2}$

Solving for TB:

$\frac{2vT}{3} = S \rightarrow T_B \frac{3S}{2v}$

Comparison of Results

Anton's Total Time: $T_A = \frac{5S}{3v}$

Boris's Total Time: $T_B = \frac{3S}{2v}$

Time Difference:

$\Delta t = T_A - T_B = \frac{5S}{3v} - \frac{3S}{2v} = \frac{10S - 9S}{6v} = \frac{S}{6v}$

$v_S = ?$

$185.$

Conclusion:

Boris finishes the race first, with a time advantage of $\Delta t = \frac{S}{6v}$

Key Reason: Boris's strategy optimizes acceleration and deceration phases by allocating time proportionally, whereas Anton's fixed-distance phases prolong his acceleration/deceleration durations.

2. Define the Problem:

Two jugglers throw balls with the same initial velocity v_0 at angles θ_1 and θ_2 respectively.

The distance between the jugglers is S

The balls are thrown and caught at the same height

Ignore air resistance.

Equations of Motion:

For the ball thrown by the first juggler (angle θ_1):

$$\begin{cases} x_1(t) = v_0 \cos\theta_1 \cdot t \\ y_1(t) = v_0 \sin\theta_1 \cdot t - \frac{1}{2} g t^2 \end{cases}$$

For the ball thrown by the second juggler (angle θ_2, thrown to the left)

$$\begin{cases} x_2(t) = S - v_0 \cos\theta_2 \cdot t \\ y_2(t) = v_0 \sin\theta_2 \cdot t - \frac{1}{2} g t^2 \end{cases}$$

Distance Between the Balls:

The square of the distance $D^2(t)$ between the two balls at time t is:

$$D^2(t) = [S - v_0 t(\cos\theta_1 + \cos\theta_2)]^2 + [v_0 t(\sin\theta_1 - \sin\theta_2)]^2$$

Minimize the Distance:

To find the minimum distance, take the derivative of $D^2(t)$ with respect to t and set it to zero:

$$t = \frac{S(\cos\theta_1 + \cos\theta_2)}{v_0[(\cos\theta_1 + \cos\theta_2)^2 + (\sin\theta_1 - \sin\theta_2)^2]}$$

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Substitute t Back:

Substitute this t back into $D^2(t)$ to find the minimum distance squared:

$$D^2_{min} = S^2 \cdot \frac{(\sin\theta_2 - \sin\theta_1)^2}{(\cos\theta_1 + \cos\theta_2)^2 + (\sin\theta_1 - \sin\theta_2)^2}$$

Simplify Using Trigonometric Identities:

Use the trigonometric identities:

$$\sin\theta_2 - \sin\theta_1 = 2\cos\left(\frac{\theta_1+\theta_2}{2}\right)\sin\left(\frac{\theta_1-\theta_2}{2}\right)$$

$$\cos\theta_1 + \cos\theta_2 = 2\cos\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right)$$

Simplify to find:

$$D_{min} = S \cdot \left|\sin\left(\frac{\theta_1-\theta_2}{2}\right)\right|$$

Conclusion:

The minimum distance between the balls during their flight is:

$$D_{min} = S \cdot \left|\sin\left(\frac{\theta_1-\theta_2}{2}\right)\right|$$

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3. Identify the Processes

$1\to2$: Isochoric heating (Volume constant, temperature and pressure increase)

$2\to3$: Isothermal expansion (temperature constant, pressure decreases, Volume ~~decreases)~~ increases)

$3\to4$: Isochoric cooling (Volume constant, temperature and pressure decrease)

$4\to1$: Isothermal compression (temperature constant, pressure increases, Volume decreases)

Calculate Heat and Work for Each Process.

$1\to2$: Isochoric heating, $Q_{1-2} = nC_V\Delta T$

Given $C_V = \frac{3}{2}R$ and $\Delta T = 8R$, so $Q_{1-2} = 12nR$

$2\to3$: Isothermal expansion, $Q_{2-3} = W_{2\to3} = nRT\ln(\frac{V_3}{V_2})$

Given the volume increases by a factor of 3, so $Q_{2-3} = 12nR\ln3$

$3\to4$ Isochoric cooling, $Q_{3-4} = -nC_V\Delta T = -8nR$

$4\to1$ Isothermal compression, $Q_{4-1} = W_{4-1} = -nRT\ln(\frac{V_1}{V_4})$

Given the volume decreases by a factor of 3, so $Q_{4-1} = -4nR\ln3$. ✓

Calculate Net Work Done:

Net work done, $W_{net} = W_{2-3} + W_{4-1} = 12nR\ln3 - 4nR\ln3 = 8nR\ln3$

Calculate Total Heat Absorbed:

Total heat absorbed, $Q_H = Q_{1-2} + Q_{2-3} = 12nR + 12nR\ln3$

Calculate Efficiency:

Efficiency: $\eta = \frac{W_{net}}{Q_H} = \frac{8nR\ln3}{12nR+12nR\ln3} = \frac{8\ln3}{12(1+\ln3)} = \frac{2\ln3}{3(1+\ln3)}$

Numerically, $\ln3 \approx 1.0986$, so $\eta = \frac{2\times1.0986}{3\times2.0986} = \frac{2.1972}{6.2972} \approx 0.349\ (34.9\%)$.

Rounding to the nearest integer, the efficiency is 35%

15б

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Express charges in Terms of Potentials:

Potential at distance d from charge q is $\phi = K\frac{q}{d}$

Charges q_1, q_2, q_3 can be expressedas:

$q_1 = \frac{\phi_1 d}{K}$, $q_2 = \frac{\phi_2 d}{K}$, $q_3 = \frac{\phi_3 d}{K}$

Forces on the First Ball:

Using Coulomb's law, the force between two charges q_i and q_j separated by distance d is: $F_{ij} = K\frac{q_i q_j}{d^2}$

Applying this to the forces on the first ball: $F_{12} = K\frac{q_1 q_2}{d^2} = \frac{\phi_1\phi_2}{K}$, $F_{13} = \frac{\phi_1\phi_3}{K}$

Vector Addition of Forces

The angle between F_{12} and F_{13} is $60°$

The magnitude of the resultant force F_1 is given by:

$F_1 = \sqrt{F_{12}^2 + F_{13}^2 + 2F_{12}F_{13}\cos(60°)}$

Simplifying the expression: $F_1 = \frac{\phi_1}{K}\sqrt{\phi_2^2 + \phi_3^2 + \phi_2\phi_3}$

Final Answer:

$F_1 = \frac{\phi_1}{K}\sqrt{\phi_2^2 + \phi_3^2 + \phi_2\phi_3}$

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Calculate Air Density and Buoyant Force change:

Calculate Air Density Before and After Humidity change.

Initial Conditions (10% Relative Humidity):

Vapor Pressure: $0.1 \times 9000Pa = 900Pa$

Dry Air Pressure: $100000Pa - 900Pa = 99100Pa$

Density of Dry Air: $\frac{99100Pa \times 0.02897}{8.314 \times 316.15} \approx 1.092 kg/m^3$

Density of Water Vapor: $\frac{900 \times 0.018015}{8.314 \times 316.15} \approx 0.00617 kg/m^3$

Total Density: $1.092 + 0.00617 \approx 1.098 kg/m^3$

Final Conditions (90% Relative Humidity):

Vapor Pressure: $0.9 \times 9000Pa = 8100Pa$

Dry Air Pressure: $100000Pa - 8100Pa = 91900Pa$

Density of ~~Water Va~~ Dry Air: $\frac{91900 \times 0.02897}{8.314 \times 316.15} \approx 1.013 kg/m^3$

Density of Water Vapor: $\frac{8100 \times 0.018015}{8.314 \times 316.15} \approx 0.0555 kg/m^3$

Total Density: $1.013 + 0.0555 \approx 1.0685 kg/m^3$

Calculate Difference in Buoyant Force

Initial Buoyant Force: $1.098 kg/m^3 \times 5000m^3 \times 9.81 m/s^2 \approx 53906N$

Final Buoyant Force: $1.0685 kg/m^3 \times 5000 m^3 \times 9.81 m/s^2 \approx 52459N$

Difference in Buoyant Force: $53906N - 52459N \approx 1447N$

Convert Force Difference to Mass

Mass to Drop: $\frac{1447 N}{9.81 m/s^2} \approx 147.5 kg$

Using Molar Mass Method for Precise Calculation

Difference in Air Density: $0.030 kg/m^3$

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Mass Difference: 0,030kg/m³ × 5000m³ = 150kg

Conclusion:

The weight of the ballast that must be dropped is 150kg

10б

влажность – ?